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# Solve Heat-Like with Variable Coefficients by the Differential Transform Method

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***Abstract:*** **The Paper Examines The Differential Transform Method (DTM) And Applies It To Solve Heat-Like Equations Using Variable Coefficients. DTM Reduces Complex Partial Differential Equations To Simple Closed Form Equations. The Method Is Used In A Number Of Engineering Applications. In This Paper, The Modalities And Process Are Examined And Then Applied To Solve Two Numerical Problems.**

***Keywords:*** **Solve Heat-Like, Differential Transform Method (DTM).**

## I. INTRODUCTION

One of the numerical methods used to solve differential equations is the differential transform method (DTM). It solves non-linear and linear value problems, yielding the exact value for the required derivative. The precise quantum of the derivative can be derived from the analytical function at any given point when the boundary is known or even it is unknown (Alquran et al., 2012). A polynomial is built for any differential equation and the analytical answer is obtained. DTM varies from the Taylor series, a high order method where computation of the required derivatives is obtained from the functions. DTM uses an iterative method to derivative the Taylor series answers for the given differential equations (Arikoglu and Ozkol, 2007).

Heat-like equations are seen in different engineering disciplines such as heat transfer, fluid dynamics, electrical circuits and others. Other methods used for solving heat-like with variable coefficients are the Adomian decomposition and Adomian methods, variation iteration methods and others (Ayaz, 2004). As indicated earlier, DTM is based on the Taylor series, but the method is simpler. Solutions are derived as polynomials. Earlier DTM found applications for partial differential equations that had discontinuity (Chen and Ho, 1999). This paper examines methods to solve heat-like equations with variable coefficients

## II. HEAT-LIKE EQUATION

Heat equation uses partial differential equation to show how the medium such as heat or electricity evolves over a certain period. Several equations such as Brownian motion, Fokker–Planck equation, partial differential equation of Black–Scholes, Poincaré conjecture, and others are used to model the motion (Secer, 2012).

For this paper, a heat-like equation where 3D boundary values are used to present the variable coefficients. The equation is given as (Taha, 2011):

$$u_t = f(x,y,z)u_{xx} + g(x,y,z)u_{yy} + h(x,y,z)u_{zz}, \qquad (1)$$

$$0 < x < a, 0 < y < b, 0 < z < c, t > 0,$$

When the conditions of the Neumann boundary are applied, then (Odibat, 2008)

$$\begin{aligned} u_x(0,y,z,t) &= f_1(y,z,t), & u_x(a,y,z,t) &= f_2(y,z,t), \\ u_y(x,0,z,t) &= g_1(x,z,t), & u_y(x,b,z,t) &= g_2(x,z,t), \qquad (2) \\ u_z(x,y,0,t) &= h_1(x,y,t), & u_z(x,y,c,t) &= h_2(x,y,t), \end{aligned}$$

Definition of the initial condition is

$$u(x,y,z,0) = \varphi(x,y,z)\ . \qquad (3)$$

## III. N-DIMENSIONAL DIFFERENTIAL TRANSFORM

The N dimensional method helps to solve equations with any restriction or discretization and any approximation or rounding off is not done. A function to which differential transform is applied is (Tabatabaei et al., 2012):

$$W(k_1,k_2,\dots,k_n) = \frac{1}{k_1!k_2!\dots k_n!}\left[\frac{\partial^{k_1+k_2+\dots+k_n}\ w(x_1,x_2,\dots,x_n)}{\partial x_1^{k_1}\,\partial x_2^{k_2}\dots\,\partial x_n^{k_n}}\right]_{x_1=0,x_2=0,\dots,x_n=0} . \qquad (4)$$

In equation (4), the original function is $w(x_1,x_2,\dots,x_n)$ and the transformed function is $W(k_1,k_2,\dots,k_n)$ . For the transformed function, the differential inverse transform of the transformed function is given as:

$$w(x_1,x_2,\dots,x_n) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\dots\sum_{k_n=0}^{\infty} W(k_1,k_2,\dots,k_n)\,x_1^{k_1}\,x_2^{k_2}\dots x_n^{k_n}. \qquad (5)$$

In real life use for engineering applications, the differential transform of function $w(x_1,x_2,\dots,x_n)$ is given as a finite series. Therefore, equation (5) now becomes.

$$w(x_1,x_2,\dots,x_n) = \sum_{k_1=0}^{m_1}\sum_{k_2=0}^{m_2}\dots\sum_{k_n=0}^{m_n} W(k_1,k_2,\dots,k_n)\,x_1^{k_1}\,x_2^{k_2}\dots x_n^{k_n}. \qquad (6)$$

## IV. NUMERICAL EXAMPLE

This section presents an analysis and solution of numerical problems for 1D and 2D models (Tabatabaei et al., 2012).

**Solution for one dimensional model:**

The following model is considered (Tabatabaei et al., 2012):

$$u_t = \frac{1}{2}x^2 u_{xx}, \quad 0 < t < 1\ ,\ t > 0\ , \qquad (7)$$

When the following boundary conditions are applied

$$u(0,t) = 0, \quad u(1,t) = e^t, \qquad (8)$$

With the following initial condition

$$u(x,0) = x^2\ . \qquad (9)$$

Considering the value of equation (7), the transform is:

$$U(k,h+1) = \frac{1}{2(h+1)}\sum_{r=0}^{k}\sum_{s=0}^{h}\delta(r-2,h-s)\,(k-r+2)\,(k-r+1)\,U(k-r+2,s), \qquad (10)$$

For the values given in (8), the differential transform is applied to get:

$$U(0,h)=0, \quad U(1,h)=\frac{1}{h!}. \tag{11}$$

$$U(k,0)=\delta(k-2)=\begin{cases}1 & k=2\,,\\ 0 & k\neq 2\,.\end{cases} \tag{12}$$

In equations (11) and (12), the values of k, h are substituted in (9), then using recursive process, $U(k,h)$ are written as:

$$U(k,h+1)=0, \quad k=0,1,3,4,5,...., \quad h=0,1,2,3,... \quad ,U(2,h)=\frac{1}{h!} \tag{13}$$

In equation (6), the inverse transformation method for 2D is applied and the answer is:

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U(k,h)x^k t^h = U(2,0)\,x^2t^0+U(2,1)\,x^2t^1+U(2,2)\,x^2t^2+U(2,3)\,x^2t^3 \\ &\quad +U(2,4)\,x^2t^4+...+U(2,n)\,x^nt^n. \end{aligned} \tag{14}$$

The answer for the closed form is given as:

$$\begin{aligned} u(x,t) &= x^2+x^2t^1+\tfrac{1}{2!}x^2t^2+\tfrac{1}{3!}x^2t^3+\tfrac{1}{4!}x^2t^4+\tfrac{1}{5!}x^2t^5+... \\ &= x^2(1+t^1+\tfrac{1}{2!}t^2+\tfrac{1}{3!}t^3+\tfrac{1}{4!}t^4+\tfrac{1}{5!}t^5+...)=x^2\,e^t\,. \end{aligned} \tag{15}$$

**Solution for two-dimensional model:**

The 2D model equation is given below (Tabatabaei et al., 2012):

$$u_t=\frac{1}{2}(y^2\,u_{x\,x}+x^2\,u_{y\,y}), \quad 0<x\ ,\ y<1\ ,\ t>0 \quad , \tag{16}$$

Neumann boundary conditions apply to the equation:

$$\begin{aligned} u_x(0,y,t)&=0, \qquad u_x(1,y,t)=2\,\sinh t\ ,\\ u_y(x,0,t)&=0, \qquad u_y(x,1,t)=2\,\cosh t\ , \end{aligned} \tag{17}$$

The initial conditions considered are

$$u(x,y,0)=y^2. \tag{18}$$

Considering the 2D transform given in (16), the following equation is derived:

$$\begin{aligned} U(k,h,m+1) &= \tfrac{1}{2(m+1)}\Bigg[\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{l=0}^{m}\delta(r,s-2,m-l)\,(k-r+1)\,(k-r+2)\,U(k-r+2,h-s,l) \\ &\quad +\sum_{r=0}^{k}\sum_{s=0}^{h}\sum_{l=0}^{m}\delta(r-2,s,m-l)\,(h-s+1)\,(h-s+2)\,U(k-r,h-s+2,l)\Bigg]\ , \end{aligned} \tag{19}$$

Considering the boundary conditions given in (17), the equation becomes:

$$U(1,h,m)=0, \qquad U(2,h,m)=\begin{cases} 0 & \text{for } m \text{ is even}, \\ \frac{2}{m!} & \text{for } m \text{ is odd}; \end{cases} \tag{20}$$

$$U(k,1,m)=0, \qquad U(k,2,m)=\begin{cases} \frac{2}{m!} & \text{even } m, \\ 0 & \text{odd } m. \end{cases} \tag{21}$$

The initial condition given in equation (18), the equation now becomes:

$$U(k,h,0)=\delta(k,h-2,m)=\begin{cases} 1 & k=m=0 \text{ and } h=2, \\ 0 & \text{otherwise}. \end{cases} \tag{22}$$

Values of k and h are replaced in equation (20), (21) and (22) into the values given in equation (19), $U(k,h)$ are calculated as:

$$\begin{aligned} &U(k,h,m+1)=0, \quad \text{for } k=1,3,4,5,6,7,\ldots,\ h=0,1,2,3,\ldots,\ and\ m=0,1,2,3,\ldots, \\ &U(0,h,m+1)=0, \quad \text{for } h=0,1,3,4,\ldots \quad \text{and} \quad m=0,1,2,3,\ldots, \\ &U(2,h,m+1)=0, \quad \text{for } h=1,2,3,4,\ldots \quad \text{and} \quad m=0,1,2,3,\ldots, \end{aligned}$$

$$\begin{aligned} U(0,2,m)&=\begin{cases} \frac{2}{m!} & \text{even } m \\ 0 & \text{odd } m, \end{cases} \\ U(2,0,m)&=\begin{cases} 0 & \text{even } m \\ \frac{2}{m!} & \text{odd } m. \end{cases} \end{aligned} \tag{23}$$

The rule of inverse transform is applied to the 3D for values given in (6). the values obtained are:

$$\begin{aligned} u(x,y,t) &= \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\sum_{m=0}^{\infty} x^k y^h t^m U(k,h,m) = 2y^2 + y^2t^2 + \tfrac{2}{4!}y^2t^4 + \tfrac{2}{6!}y^2t^6 + \cdots \\ &\quad +2x^2t^1 + \tfrac{2}{3!}x^2t^3 + \tfrac{2}{5!}x^2t^5 + \tfrac{2}{7!}x^2t^7 + \cdots . \end{aligned} \tag{24}$$

When the above equation is simplified, the closed form of the solution is:

$$\begin{aligned} u(x,y,t) &= 2y^2\left(1+\tfrac{t^2}{2!}+\tfrac{t^4}{4!}+\tfrac{t^6}{6!}+\cdots\right) + 2x^2\left(t^1+\tfrac{t^3}{3!}+\tfrac{t^5}{5!}+\tfrac{t^7}{7!}+\cdots\right) \\ &= 2y^2\cosh t + 2x^2\sinh t\,. \end{aligned} \tag{25}$$

## V. CONCLUSION

The paper studied and used the differential transform method to solve the problems of heat-like equations with variable coefficients. The theory, application and use of DTM was examined. Two problems related to one-dimensional and two-dimensional heat-like equations were solved. It is seen that in both cases, the complex problems were resolved. In both instances, the problems were resolved to form closed form of the solution and these can be used to solve problems in real-life applications.

## REFERENCES

[1] Arikoglu, A. and Ozkol, I., 2007. Solution of fractional differential equations by using differential transform method. *Chaos, Solutions & Fractals*, 34(5), pp.1473-1481.

[2] Ayaz, F., 2004. Solutions of the system of differential equations by differential transform method. *Applied Mathematics and Computation*, 147(2), pp.547-567.

[3] Chen, C.O.K. and Ho, S.H., 1999. Solving partial differential equations by two-dimensional differential transform method. *Applied Mathematics and computation*, 106(2-3), pp.171-179.

[4] Odibat, Z. M. 2008. Differential transform method for solving Volterra integral equation with separable kernels. *Mathematical and Computer Modelling*, 48 (7-8), pp. 1144-1149.

[5] Secer, A., 2012. Approximate analytic solution of fractional heat-like and wave-like equations with variable coefficients using the differential transforms method. *Advances in Difference Equations*, 2012(1), p.198.

[6] Tabatabaei, K., Celik, E. & Tabatabaei, R. 2012. The differential transform method for solving heat-like with variable coefficients. *Turkey Journal of Physics*, 36 (2012), 87-98.

[7] Taha, B.A., 2011. The use of reduced differential transform method for solving partial differential equations with variable coefficients. *Journal of Basrah Researches (Sciences)*, 37(4).